

# Conditional Spin Counting Statistics as a Probe of Coulomb Interaction

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Full counting statistics is a powerful tool to characterize the noise and correlations in transport through mesoscopic systems. In this work, we propose the concept of conditional spin counting statistics, i.e., the statistical fluctuations of a spin up (down) current given the observation of a given spin down (up) current. In the context of transport through a single quantum dot, it is demonstrated that a strong Coulomb interaction leads to a conditional spin counting statistics that exhibits a substantial change in comparison to that without Coulomb interaction. It thus can be served as an effective way to probe the Coulomb interactions in mesoscopic transport systems.

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## I. INTRODUCTION

The exploration of full counting statistics (FCS) in mesoscopic systems has vital roles to play in gaining deep insight into microscopic mechanisms of transport and temporal correlations between charge carriers which are not available from conventional measurements of time-averaged current alone [1–3]. Particularly, recent advances in nanotechnology have made it possible to measure electron transport processes that take place at single-electron level [4–12]. All statistical cumulants of the number of transferred particles can now be extracted experimentally.

Theoretical study of FCS based on the scattering approach turns out to be very powerful for characterizing statistics of noninteracting electron transport through a variety of systems [3, 13–15]. Current fluctuations would satisfy the Poissonian process if the tunneling events were statistically independent. Normally, electric current noise is suppressed from the Poisson value due to the Pauli exclusion principle which forbids two electrons with the same quantum states to be superimposed [16, 17]. On the other hand, Coulomb interaction also serves as another important mechanism that can correlate wave packets. Particularly, with continued miniaturization of the system size, electron interaction effects become increasingly important in mesoscopic transport [18, 19]. Recent study shows that the noise is indeed more sensitive to interactions than the average current [20–25]. Depending on different physical regimes concerned, the Coulomb repulsions may either decrease or increase noise correlations [26–32]. However, for a given mesoscopic system, the effects on the noise features due to Fermi statistics and that due to Coulomb interactions are normally intimately mixed.

It is clear that the Pauli principle acts only on electrons with the same spin, while electrons with opposite spins are only correlated by the Coulomb interaction. It thus

inspires us to propose in this work the concept of conditional spin counting statistics: The statistical fluctuations of a spin up (down) current given the observation of a given spin down (up) current. The theory is applied to a simple system—a single quantum dot (QD) tunnel-coupled to two normal electrodes. Although the net current is not spin polarized, the transport of up and down spins are actually correlated to each other via Coulomb interaction. It is demonstrated that the strong Coulomb interaction leads to a conditional spin counting statistics that undergoes a substantial change in comparison to that without Coulomb interaction. It thus may offer an effective and transparent way to probe the Coulomb interactions in various mesoscopic transport systems.

The paper is organized as follows. In section II, we describe the single QD system in different bias configurations, such that the effectiveness of the Coulomb interactions can be fully taken into account. Section III is dedicated to the theory of conditional spin counting statistics. Application of the theory to QD system is demonstrated in Section IV, with focus on its effectiveness in probing Coulomb interaction. It is then followed by the conclusion in section V.

## II. THE MODEL

We consider electron transport through a single QD with Coulomb interaction, as schematically shown in Fig. 1. The entire system is described by the Hamiltonian  $H = H_B + H_{QD} + H'$ , with

$$H_{el} = \sum_{\alpha=L,R} \sum_{k\sigma} \epsilon_{\alpha k} c_{\alpha k\sigma}^\dagger c_{\alpha k\sigma}, \quad (1a)$$

$$H_{QD} = \sum_{\sigma} \epsilon_{\sigma} d_{\sigma}^\dagger d_{\sigma} + U n_{\uparrow} n_{\downarrow}, \quad (1b)$$

$$H' = \sum_{\alpha=L,R} \sum_{k\sigma} (t_{\alpha k} c_{\alpha k\sigma}^\dagger d_{\sigma} + \text{h.c.}). \quad (1c)$$

Here  $H_B$  models the noninteracting electrons in the left ( $\alpha=L$ ) and right ( $\alpha=R$ ) electrodes, with  $c_{\alpha k\sigma}^\dagger$  ( $c_{\alpha k\sigma}$ ) the electron creation (annihilation) operator in the corre-

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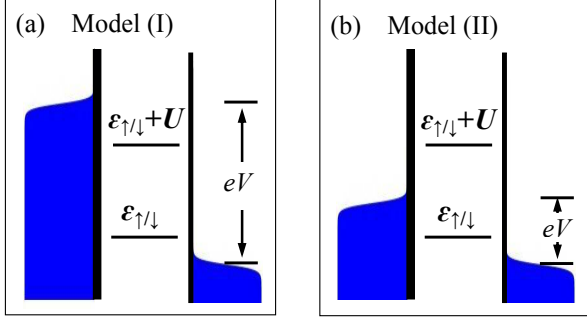


FIG. 1: Schematic setup of transport through a single quantum dot in different bias configurations. Model (I): The bias is large enough to overcome the Coulomb interaction. Model (II): A small bias is applied across the QD. The Coulomb repulsion allows at most one electron (either spin up or spin down) to be occupied. In both cases, the Fermi functions can be approximated by either one or zero, so the temperature is not involved here.

sponding electrode. The electron distributions in the electrodes are governed by the electrochemical potentials  $\mu_L$  and  $\mu_R$ . In this simple model, we choose a symmetric bias voltage, such that  $\mu_L = eV/2$  and  $\mu_R = -eV/2$ .  $H_{QD}$  describes the QD with one spin-degenerate energy level  $\varepsilon_\sigma$  and the Coulomb interaction  $U$  on the dot, where  $n_\sigma = d_\sigma^\dagger d_\sigma$  is the occupation operator, with  $d_\sigma^\dagger$  ( $d_\sigma$ ) the electron annihilation (creation) operator in the QD. Electron tunneling between electrodes and QD is depicted by  $H'$ . The tunnel-coupling strength is characterized by the intrinsic tunneling width  $\Gamma_\alpha(\omega) = 2\pi g_\alpha(\omega)|t_\alpha|^2$ , where  $g_\alpha(\omega)$  is the density of states in the electrode  $\alpha$ . Hereafter we assume flat bands in the electrodes, which yields energy-independent couplings  $\Gamma_\alpha$ . Throughout this work, we set  $\hbar = 1$  for the Planck constant, unless stated otherwise.

By specifying which excitation energies lie within the energy window defined by the Fermi levels  $\mu_L$  and  $\mu_R$ , the following two configurations will be considered. Model (i): The bias is large enough to overcome the Coulomb interaction, as schematically shown in Fig. 1(a). Model (ii): Only the single level  $\varepsilon_{\uparrow/\downarrow}$  is within the bias, as shown in Fig. 1(b). The charge transport is strongly correlated. The present model is simple; however, we will show that it is adequate to address the essence of the conditional spin counting statistics and its effectiveness in probing Coulomb interaction.

### III. FULL COUNTING STATISTICS VERSUS CONDITIONAL SPIN COUNTING STATISTICS

The FCS of charge transfer can be described in the framework of counting field-dressed approach, in which electron jumps between reduced system and electrodes are taken into account by introducing a corresponding

counting field  $\chi$ . This results in a  $\chi$ -resolved quantum master equation, which in the Born-Markov limit can be formally written as [33–37]  $\frac{\partial}{\partial t}\rho(\chi, t) = \mathcal{L}_\chi\rho(\chi, t)$ . The cumulant generating function (CGF) is determined from the lowest eigenvalue of  $\mathcal{L}_\chi$ . For instance, the CGF for model (I) is given by [37]

$$\mathcal{F}(\chi) = -t_c\{\Gamma_L + \Gamma_R - \sqrt{(\Gamma_L - \Gamma_R)^2 + 4\Gamma_L\Gamma_R e^{-i\chi}}\}, \quad (2a)$$

while for model (II), the CGF reads

$$\mathcal{F}(\chi) = -\frac{t_c}{2}\{2\Gamma_L + \Gamma_R - \sqrt{(2\Gamma_L - \Gamma_R)^2 + 8\Gamma_L\Gamma_R e^{-i\chi}}\}, \quad (2b)$$

where  $t_c$  is the counting time. Apart from an effective doubling of the rate  $\Gamma_L$ , the result for model (II) [Eq. (2b)] is qualitatively similar to that of model (I) [Eq. (2a)]. Therefore the charge counting statistics is not the best possible probe of interactions. We will show later, on the contrary, the behavior of conditional spin current noise is completely different, and can be served as a sensitive diagnostic tool to probe the Coulomb interaction.

Now we introduce the concept of conditional spin counting statistics: The statistical fluctuations of the spin- $\uparrow$  ( $\downarrow$ ) current given the observation of a given spin- $\downarrow$  ( $\uparrow$ ) current. These statistics may be calculated from the conditional distribution functions,  $P(I_\uparrow|I_\downarrow)$  or  $P(I_\downarrow|I_\uparrow)$ , the probability of observing one spin current component given an observation of the other. To obtain the mixed generating functions of conditional spin counting statistics, we introduce spin-resolved counting fields  $\chi_\uparrow$  and  $\chi_\downarrow$ , which are used to characterize, respectively, jumps of up and down spins through a specific junction. The corresponding  $(\chi_\uparrow, \chi_\downarrow)$ -resolved master equation can be formally written as  $\frac{\partial}{\partial t}\rho(\chi_\uparrow, \chi_\downarrow, t) = M(\chi_\uparrow, \chi_\downarrow)\rho(\chi_\uparrow, \chi_\downarrow, t)$ . In the stationary limit, the counting time  $t_c$  is much longer than the time of tunneling through the system, the joint generating function  $\mathcal{F}(\chi_\uparrow, \chi_\downarrow)$  and consequently the counting statistics is determined from the minimal eigenvalue of  $M$  according to

$$\mathcal{F}(\chi_\uparrow, \chi_\downarrow) = -\lambda_{\min}(\chi_\uparrow, \chi_\downarrow)t_c, \quad (3)$$

where  $\lambda_{\min}$  satisfies  $\lambda_{\min}|_{\chi_\uparrow \rightarrow 0, \chi_\downarrow \rightarrow 0} \rightarrow 0$ .

From the definition of the joint generating function, the joint probability distribution of transmitted spins can be extracted by Fourier transforming on both variables,

$$P(N_\uparrow, N_\downarrow, t_c) = \int_0^{2\pi} \frac{d\chi_\uparrow d\chi_\downarrow}{(2\pi)^2} e^{-\lambda_{\min} t_c - i(N_\uparrow \chi_\uparrow + N_\downarrow \chi_\downarrow)}. \quad (4)$$

Replacing  $N_\uparrow = I_\uparrow t_c$  and  $N_\downarrow = I_\downarrow t_c$  gives the joint probability distribution of the two currents. In the stationary limit ( $t_c \gg \Gamma_L^{-1}, \Gamma_R^{-1}$ ), it is justified to evaluate the integral in the saddle point approximation [37–40]. The dominant term contributing to the joint distribution is then given by

$$P(I_\uparrow, I_\downarrow, t_c) = -t_c \min_{\chi_\uparrow, \chi_\downarrow} \{\lambda_{\min} + iI_\uparrow \chi_\uparrow + iI_\downarrow \chi_\downarrow\}. \quad (5)$$

The mixed generating functions of conditional spin counting statistics may be calculated by only Fourier transforming on one of the above variables in Eq. (5). For instance, integrating over the  $\chi_\uparrow$  variable yields  $\mathcal{F}(\chi_\uparrow, I_\downarrow)$ . Due to the Bayes's theorem, which relates the joint distribution and conditional distribution functions  $P(I_\uparrow, I_\downarrow) = P(I_\uparrow|I_\downarrow)P(I_\downarrow) = P(I_\downarrow|I_\uparrow)P(I_\uparrow)$ , the mixed generating function then is given by  $\mathcal{F}(\chi_\uparrow|I_\downarrow) = \mathcal{F}(\chi_\uparrow, I_\downarrow) - \mathcal{F}(0, I_\downarrow)$ . The statistical fluctuations of spin  $\uparrow$  current given the observation of a given spin  $\downarrow$  current can be obtained from  $\mathcal{F}(\chi_\uparrow|I_\downarrow)$  by performing derivatives with respect to the counting field

$$\langle I_\uparrow^k \rangle_{I_\downarrow} = -\frac{(-i\partial_{\chi_\uparrow})^k}{t_c} \mathcal{F}(\chi_\uparrow|I_\downarrow)|_{\chi_\uparrow \rightarrow 0}. \quad (6)$$

Analogously, the conditional spin counting statistics of spin  $\downarrow$  current can be evaluated from  $\mathcal{F}(\chi_\downarrow|I_\uparrow)$ .

#### IV. RESULTS AND DISCUSSION

First let us focus on the model (I), as schematically shown in Fig. 1(a). The involving states are  $|0\rangle$ -empty dot,  $|\uparrow/\downarrow\rangle$ -occupation by a spin- $\uparrow/\downarrow$  electron, and  $|d\rangle$ -doubly occupied. The  $(\chi_\uparrow, \chi_\downarrow)$ -resolved quantum master equation reads  $\frac{\partial}{\partial t} \rho(\chi_\uparrow, \chi_\downarrow) = M_I(\chi_\uparrow, \chi_\downarrow) \rho(\chi_\uparrow, \chi_\downarrow)$ , where  $M_I$  is a  $4 \times 4$  matrix

$$M_I = \begin{pmatrix} -2\Gamma_L & e^{i\chi_\uparrow}\Gamma_R & e^{i\chi_\downarrow}\Gamma_R & 0 \\ \Gamma_L & -\Gamma_L - \Gamma_R & 0 & e^{i\chi_\downarrow}\Gamma_R \\ \Gamma_L & 0 & -\Gamma_L - \Gamma_R & e^{i\chi_\uparrow}\Gamma_R \\ 0 & \Gamma_L & \Gamma_L & -2\Gamma_R \end{pmatrix}. \quad (7)$$

The minimal eigenvalue of  $M_I$  is then determined, which leads to the joint distribution  $P_I(I_\uparrow, I_\downarrow, t_c)$  of spin  $\uparrow$  and  $\downarrow$  currents

$$\frac{\log P_I}{t_c} = -(\Gamma_L + \Gamma_R) + \sum_{\sigma=\uparrow, \downarrow} \left\{ \frac{\Omega_\sigma}{2} - I_\sigma \log \left( \frac{\Omega_\sigma I_\sigma}{\Gamma_L \Gamma_R} \right) \right\}, \quad (8)$$

with  $\Omega_\sigma = 2I_\sigma + \sqrt{4I_\sigma^2 + (\Gamma_L - \Gamma_R)^2}$ . Apparently, the joint probability factorizes  $P_I(I_\uparrow, I_\downarrow) = P(I_\uparrow)P(I_\downarrow)$ , which implies that spin  $\uparrow$  and  $\downarrow$  currents are uncorrelated. The resultant mixed generating function for conditional spin counting statistics then reads

$$\mathcal{F}_I(\chi_\uparrow|I_\downarrow) = -\frac{t_c}{2} \{ \Gamma_L + \Gamma_R - \sqrt{(\Gamma_L - \Gamma_R)^2 + 4\Gamma_L \Gamma_R e^{-i\chi_\uparrow}} \}. \quad (9)$$

which is the same as the charge counting statistics for model (I) in Eq. (2a), except for an overall factor of  $\frac{1}{2}$ . This is because the spin transport through the QD independently, and each spin component contribution 50% of the total current. The cumulants of conditional spin counting statistics are thus the same as those of unconditional one. For instance, the first and second mixed

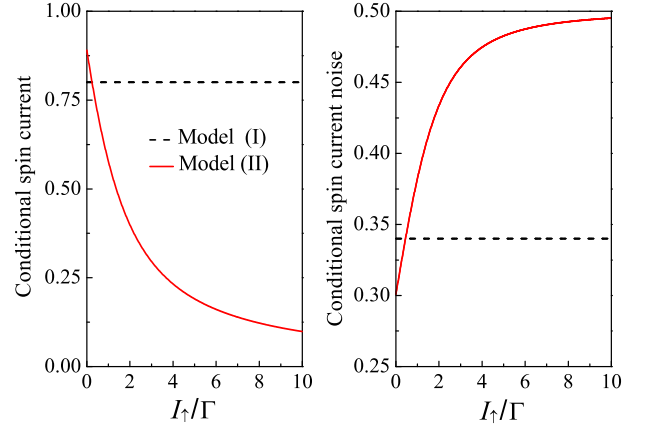


FIG. 2: Conditional spin  $\uparrow$  current and its noise versus spin  $\downarrow$  current. The dashed and solid lines correspond to results of the model (I) and (II), respectively. We use  $\Gamma_L = \Gamma$  as the energy unit, and set  $\Gamma_R = 4\Gamma$ .

cumulants are given, respectively, by

$$\langle I_\uparrow \rangle_{I_\downarrow} = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}, \quad (10a)$$

$$\frac{\langle I_\uparrow^2 \rangle_{I_\downarrow}}{2e\langle I_\uparrow \rangle_{I_\downarrow}} = \frac{\Gamma_L^2 + \Gamma_R^2}{2(\Gamma_L + \Gamma_R)^2}. \quad (10b)$$

In this case the system can be mapped onto that transport through a single level without Coulomb interaction. One may apply analogous analysis to the conditional counting statistical of the spin  $\downarrow$  current given the observation of a spin  $\uparrow$  current, and the same results as Eq. (10) will be obtained.

Let us now consider the conditional spin counting statistics for model (II) as shown in Fig. 1(b), where at most one electron can be occupied and the spin currents are strongly correlated. Thus the involving states are reduced to  $|0\rangle$ -empty dot,  $|\uparrow/\downarrow\rangle$ -occupied by a spin  $\uparrow/\downarrow$  electron. The spin-resolved quantum master equation reads  $\frac{\partial}{\partial t} \rho(\chi_\uparrow, \chi_\downarrow) = M_{II}(\chi_\uparrow, \chi_\downarrow) \rho(\chi_\uparrow, \chi_\downarrow)$ , where the  $3 \times 3$  rate matrix is given by

$$M_{II} = \begin{pmatrix} -2\Gamma_L & e^{i\chi_\uparrow}\Gamma_R & e^{i\chi_\downarrow}\Gamma_R \\ \Gamma_L & -\Gamma_R & 0 \\ \Gamma_L & 0 & -\Gamma_R \end{pmatrix}. \quad (11)$$

The minimal eigenvalue can be readily evaluated, which results in the joint probability for the spin currents for the model (II)

$$\frac{\log P_{II}}{t_c} = \frac{1}{2} (\Lambda - 2\Gamma_L - \Gamma_R) - \sum_{\sigma} I_\sigma \log \left( \frac{\Lambda I_\sigma}{\Gamma_L \Gamma_R} \right), \quad (12)$$

where  $\Lambda = 2(I_\uparrow + I_\downarrow) + \sqrt{4(I_\uparrow + I_\downarrow)^2 + (2\Gamma_L - \Gamma_R)^2}$ . The term inside the logarithm shows unambiguously that the spin  $\uparrow$  and  $\downarrow$  currents are correlated. Further integrating

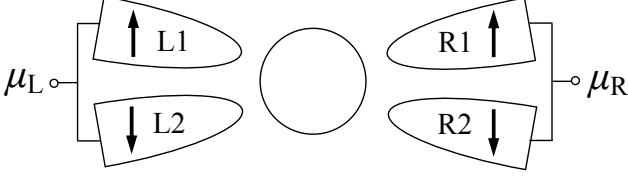


FIG. 3: Schematic setup for measuring the conditional spin counting statistics by employing a device made up of four spin-polarized terminals (see text).

over the  $\chi_\uparrow$  variable gives rise to the mixed generating function

$$\mathcal{F}_\Pi(\chi_\uparrow|I_\downarrow) = \frac{t_c}{2} \left\{ G(\chi_\uparrow) - G(0) - 2I_\downarrow \log \left[ \frac{2I_\downarrow + G(\chi_\uparrow)}{2I_\downarrow + G(0)} \right] \right\}, \quad (13)$$

with  $G(\chi_\uparrow) = \sqrt{4I_\downarrow^2 + 4e^{\chi_\uparrow}\Gamma_L\Gamma_R + (2\Gamma_L - \Gamma_R)^2}$ . The conditional spin counting statistics may be calculated via taking derivatives with respect to  $\chi_\uparrow$ . For instance, the first mixed cumulant yields the conditional spin current

$$\langle I_\uparrow \rangle_{I_\downarrow} = \frac{\Gamma_L\Gamma_R}{2I_\downarrow + G(0)}, \quad (14a)$$

and the second mixed cumulant corresponds to the conditional shot noise of spin  $\uparrow$  current

$$\frac{\langle I_\uparrow^2 \rangle_{I_\downarrow}}{2e\langle I_\uparrow \rangle_{I_\downarrow}} = \frac{1}{2} \left( 1 - \frac{2\Gamma_L\Gamma_R}{[2I_\downarrow + G(0)]G(0)} \right), \quad (14b)$$

where  $G(0) = \sqrt{4I_\downarrow^2 + 4\Gamma_L^2 + \Gamma_R^2}$ . Both show a radical change in comparison to those without Coulomb interaction, see Eq. (10) for the model (I). One may also compare the conditional spin current and noise with the unconditional quantities [41]

$$\langle I_\uparrow \rangle_{\text{uncond}} = \frac{\Gamma_L\Gamma_R}{2\Gamma_L + \Gamma_R}, \quad (15a)$$

$$\frac{\langle I_\uparrow^2 \rangle_{\text{uncond}}}{2e\langle I_\uparrow \rangle_{\text{uncond}}} = \frac{1}{2} \left( 1 - \frac{2\Gamma_L\Gamma_R}{(2\Gamma_L + \Gamma_R)^2} \right), \quad (15b)$$

which are similar to the conditional cumulants without Coulomb interaction [Eq. (10)], except for an effective doubling of  $\Gamma_L$ . Thus, the conditional spin counting statistics manifests itself unambiguously as a sensitive probe of Coulomb interaction.

The numerical results for the conditional spin  $\uparrow$  current and its noise versus spin  $\downarrow$  current are plotted in Fig. 2(a) and (b), respectively. For the model (I), the spin  $\uparrow$  and  $\downarrow$  currents are uncorrelated, thus the conditional spin  $\uparrow$  current and its noise [Eq. (10)] are independent of the spin  $\downarrow$  current, as shown by the dashed lines in Fig. 2. The conditional spin  $\uparrow$  current  $\langle I_\uparrow \rangle_{I_\downarrow}$  for the model (II) decreases monotonically with the spin  $\downarrow$  current, and tends

to zero for  $I_\downarrow \rightarrow \infty$ . It might be larger or smaller than that of model (I), depending on tunneling rates  $\Gamma_L/\Gamma_R$  and  $I_\downarrow$ . For a given spin  $\downarrow$  current  $I_\downarrow = \frac{\Gamma_L\Gamma_R}{2\Gamma_L + \Gamma_R}$ , the conditional spin current and the unconditional one coincide ( $\langle I_\uparrow \rangle_{I_\downarrow} = \langle I_\uparrow \rangle_{\text{uncond}}$ ), which corresponds to the most likely spin current configuration for non-polarized electrodes. The conditional spin current noise for the model (II) however increases with rising  $I_\downarrow$ . It reaches the maximum  $\frac{1}{2}$  as  $I_\downarrow \rightarrow \infty$ , which reflects the rare tunneling events of up spins through the QD system given an extremely large spin  $\downarrow$  current. These unique noise behavior together with their striking distinction compared to the model (I) thus can be served as a sensitive tool to probe the Coulomb interaction in mesoscopic transport.

Finally, let us propose a setup for the measurement of conditional spin counting statistics by considering a four-terminal device [41, 42], as shown in Fig. 3. The four electrodes are made up of fully spin polarized ferromagnetic metals. The left two electrodes (L1 and L2) of opposite spin polarizations are kept at the same chemical potential  $\mu_L$ . Likewise, electrodes R1 and R2 with opposite polarizations on the right have the same chemical potential  $\mu_R$ . If the junction parameters for L1 and L2 are the same, and those for R1 and R2 are identical as well, the net current transport through the QD is not spin polarized. This setup thus can be mapped onto the model that we have analyzed. In spite of non-polarized current, it is possible to measure separately the up or down spin current in each of the four electrodes, which eventually enables access to the conditional spin counting statistics. We expect that these predictions can be tested in quantum transport experiments in the near future.

## V. CONCLUSION

In summary, we have proposed the concept of conditional spin counting statistics: The statistical fluctuations of a spin up (down) current given the observation of a given spin down (up) current. By applying the theory to a simple system—a single quantum dot, we demonstrated that in the presence of a strong Coulomb interaction the conditional spin counting statistics undergoes a substantial change in comparison to that without Coulomb interaction. It thus can be served as an effective way to probe the Coulomb interactions in mesoscopic transport systems. Furthermore, the present theory may also open the possibilities to get a deep understanding of the spin-resolved bunching behavior and its connection to the noise characteristics [43].

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